

Reinterpretation with Systems of Spheres

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Abstract. Communicating agents in open environments such as the semantic web face the problem of inter-ontological ambiguity, i.e., the problem that some agent uses a (constant, role or concept) name differently than another agent. In this paper, we propose a strategy for online ambiguity resolution relying on the ideas of belief revision and reinterpretation. The data structures guiding the conflict resolution are systems of spheres, which, in particular, allow to select a resolution result amongst other potential results. The paper defines operators for (iterated) reinterpretation based on systems of spheres and shows that they fulfill some desirable set of properties (postulates).

Keywords: belief revision · spheres · ontology · ambiguity

1 Introduction

Ambiguous use of words is a typical phenomenon of natural languages (next to others such as vagueness, anaphora etc.) that may cause misunderstandings within communicating humans. Similar problems occur also within artificial agents communicating in open environments such as the semantic web. Though artificial agents usually rely on formal languages one cannot assume that they rely on the same ontology. Hence, instead of following the unrealistic aim of one ontology for all agents, agents should be equipped with an online mechanism for identifying and resolving conflicts caused by ambiguous use of symbols.

In this paper we consider the situation of two communicating agents, where a receiver agent holds an ontology and receives (one-after-one) bits of information from a sender agent, holding a different but kindred ontology. We consider a class of operators that use the idea of reinterpretation for the resolution of logical conflicts [4, 6, 2]: The meaning of the symbol as used in the ontology is changed by broadening or weakening its extension such that the conflict is resolved, and the different meanings are interrelated by bridging axioms.

The possible ways in which the receiver’s ontology could be changed has to be constrained declaratively such that only “rational” types of changes results. This idea was one of the corner-stones of the rationality postulates for revision operators as developed in the pioneering work of AGM (Alchourron, Gärdenfors, Makinson) on belief revision [1]. One rationality postulate requires that the

outcome of the change deviates only minimally from the original knowledge base. In case of AGM the knowledge base is a logically closed set of sentences called belief set, in this paper the knowledge base is an ontology.

Usually, there is more than a single “minimal” change result, hence the change has to be supported by some data structure that allows to select a unique result. In the literature different forms of such structures have been considered, e.g., epistemic entrenchment relations, preference orders over models, selections functions in partial meet operators etc. The kind of data structure that was considered in [4]—and that is also in the focus of this paper—are systems of spheres as introduced by Grove [3] and used in prototype revision in [8].

In this paper we build on the general idea of [4] using *systems of spheres* for changing concepts by reinterpretation and we extend the operators of [4] to deal with *iterated reinterpretation*. We show that (iterated) sphere-based reinterpretation operators enjoy most of the properties one would expect from a rational semantic integration operator by considering the classical AGM-postulates [1] as well as other postulates that fit the integration scenario mentioned above.

A longer version of this paper can be found at <https://tinyurl.com/y8n3p6zo>.

2 Example

Here and in the following we assume familiarity with description logics (DLs). A receiver agent is the owner of the following ontology $\mathcal{O} = \langle O, \mathcal{V}_p, \mathcal{V}_i \rangle$ where O is a set of tbox and abox axioms over $\mathcal{V}_p \cup \mathcal{V}_i$, \mathcal{V}_p is a public vocabulary, in which agents communicate, and \mathcal{V}_i is the internal vocabulary of the receiver agent.

$$O = \{Student \sqsubseteq \neg Researcher, Researcher(peter)\} \quad \alpha = Student(peter)$$

O says that no student is a researcher and that Peter is a researcher. The information α , stemming from a trustworthy sender, has to be integrated into O . It says that Peter is a student. Information α leads to a logical conflict with the ontology. And hence a change of the ontology is triggered.

Reinterpretation traces back the conflict between O and α to different readings of the concept symbol *Student* or the constant *peter*. We consider only the reinterpretation of concept symbols, hence *Student* has to be reinterpreted.

The outcome of sphere-based reinterpretation is given in the following:

$$O \odot_{\mathbb{S}} \alpha = \{Student' \sqsubseteq \neg Researcher, Researcher(peter)\} \cup \tag{1}$$

$$\{Student' \sqsubseteq Student\} \cup \tag{2}$$

$$\{Student \sqsubseteq Student' \sqcup Researcher\} \cup \tag{3}$$

$$\{Student(peter)\} \tag{4}$$

As the receiver trusts the sender, it adopts the sender’s reading of “student” and hides its own reading in the internal vocabulary as *Student'*. As the notions are assumed to be similar, they are related by two bridging axioms: the first (line (2)) is an upper bound for *Student'*, stating that *Student'* is a subconcept of *Student*. The second one (line (3)) is an upper bound for *Student*.

In order to motivate the second bound let us write it the equivalent form $\{Student \sqcap \neg Researcher \sqsubseteq Student'\}$. This axiom says that a *Student* (student in the sender's sense) is a *Student'* (student in the receiver's sense) except for the case that it is also a researcher. The concept *Researcher* which expresses the exception and hence the difference between *Student* and *Student'* is found by exploiting the ontology for a compatible conceptual representation for the constant *peter* involved in the conflict. In order to find this conceptual representation the reinterpretation operator extracts the *most specific concept* $msc_O(peter)$ for *peter* and then does a form of concept contraction based on spheres: it weakens the original concept *Student* in the ontology such that it does not contain one of the conflicting properties of *peter*, mentioned in $msc_O(peter)$, anymore. That is, the student concept is contracted with the negation of $msc_O(peter)$. The result of this is exactly the upper bound $Student' \sqcup Researcher$ for *Student*.

3 Revision and Contraction of Concepts

The reinterpretation operators considered in this paper are based on the revision and contraction of (atomic or complex) DL concepts as defined in [4].

Let $\mathcal{O} = \langle O, \mathcal{V}_p, \mathcal{V}_i \rangle$ be an ontology. Let $\mathcal{V}_{rel} \subseteq \mathcal{V}_i \cup \mathcal{V}_p$ be a subset of the whole vocabulary, called the relevant vocabulary. It is possible to define a Tarskian consequence operator $C_{\mathcal{O}}^{\uparrow} = (C)_{\mathcal{O}, \mathcal{V}_{rel}}^{\uparrow}$ on the set of concepts C over $conc(\mathcal{V}_{rel})$. (See long version of this paper.) A set $X \subseteq conc(\mathcal{V}_{rel})$ is called *consistent* iff $\perp \notin X$. $X \subseteq conc(\mathcal{V}_{rel})$ is *maximally* $(\mathcal{O}, \mathcal{V}_{rel})$ -*consistent* iff X is consistent, $(\mathcal{O}, \mathcal{V}_{rel})$ -closed and inclusion maximal with this property. The set of maximally $(\mathcal{O}, \mathcal{V}_{rel})$ -consistent sets X is denoted $M_{\mathcal{O}, \mathcal{V}_{rel}}$. Let $M_{\mathcal{O}} = M_{\mathcal{O}, (\mathcal{V}(\mathcal{O}) \cap \mathcal{V}_i) \cup \mathcal{V}_p}$. Intuitively, $M_{\mathcal{O}}$ denotes the set of all “possible objects” in ontology \mathcal{O} . The “dynamics” of possible objects under changing axioms, vocabularies, resp. are captured by the following propositions.

Proposition 1. *Let $\mathcal{O}_1 = \langle O_1, \mathcal{V}_p, \mathcal{V}_i \rangle$, $\mathcal{O}_2 = \langle O_2, \mathcal{V}_p, \mathcal{V}_i \rangle$ and $\mathcal{V} \subseteq \mathcal{V}_p \cup \mathcal{V}_i$. Then $O_1 \subseteq O_2$ entails $M_{\mathcal{O}_2, \mathcal{V}} \subseteq M_{\mathcal{O}_1, \mathcal{V}}$.*

Proposition 2. *Let $\mathcal{O} = \langle O, \mathcal{V}_p, \mathcal{V}_i \rangle$ be an ontology, $\mathcal{V}_1, \mathcal{V}_2 \subseteq \mathcal{V}_p \cup \mathcal{V}_i$ be vocabularies and assume that the consequence operator $(\cdot)_{\mathcal{O}, \mathcal{V}_i \cup \mathcal{V}_p}^{\uparrow}$ fulfills the interpolation property. If $\mathcal{V}_1 \subseteq \mathcal{V}_2$, then for the injective function $F_{\mathcal{O}, \mathcal{V}_1, \mathcal{V}_2} : M_{\mathcal{O}, \mathcal{V}_1} \xrightarrow{inj} \mathcal{P}(M_{\mathcal{O}, \mathcal{V}_2}); X \mapsto F_{\mathcal{O}, \mathcal{V}_1, \mathcal{V}_2}(X) = \{Y \in M_{\mathcal{O}, \mathcal{V}_2} \mid Y \supseteq X\}$ it holds that $M_{\mathcal{O}, \mathcal{V}_2} = \biguplus_{X \in M_{\mathcal{O}, \mathcal{V}_1}} F_{\mathcal{O}, \mathcal{V}_1, \mathcal{V}_2}(X)$.*

For concept representation \underline{C} , i.e., a set of concepts, let $[\underline{C}]^{\mathcal{O}} = \{X \in M_{\mathcal{O}} \mid \underline{C} \subseteq X\}$ be the set of possible objects $X \in M_{\mathcal{O}}$ that are not in conflict with \underline{C} . This adapts Grove's model bracket [3]. For a concept C we let $[C]^{\mathcal{O}}$ abbreviate $[C_{\mathcal{O}}^{\uparrow}]^{\mathcal{O}}$. With this machinery we recapitulate the notion of a system of spheres of [4].

Definition 1 ([4]). *For an ontology $\mathcal{O} = \langle O, \mathcal{V}_p, \mathcal{V}_i \rangle$ and a subset $\mathcal{W} \subseteq M_{\mathcal{O}}$ a family of sets $\mathcal{S} \subseteq \mathcal{P}(M_{\mathcal{O}})$ is called a system of spheres (for short SoS) for \mathcal{W} in \mathcal{O} iff the following conditions are fulfilled: 1. \mathcal{S} is totally ordered w.r.t. set*

inclusion; 2. \mathcal{W} is inclusion minimal in \mathcal{S} ; 3. $M_{\mathcal{O}}$ is inclusion maximal in \mathcal{S} ; for all concepts C the following holds: If there is a sphere $S \in \mathcal{S}$ with $[C]^{\mathcal{O}} \cap S \neq \emptyset$, then there is an inclusion minimal sphere $S_{min} \in \mathcal{S}$ with $[C]^{\mathcal{O}} \cap S_{min} \neq \emptyset$. Let $c_{\mathcal{S}}$ denote the function that selects for each $[C]^{\mathcal{O}}$ the minimal sphere with non-empty intersection with $[C]^{\mathcal{O}}$ (which must exist due to condition 4.) One sets $c_{\mathcal{S}}(\emptyset) = M_{\mathcal{O}}$. Furthermore, let $f_{\mathcal{S}}([C]^{\mathcal{O}}) = c_{\mathcal{S}}([C]^{\mathcal{O}}) \cap [C]^{\mathcal{O}}$.

For each SoS \mathcal{S} one can define a dual chain of concept representations $\mathcal{T}_{\mathcal{S}} = \{\bigcap S \mid S \in \mathcal{S}\}$. Because in the following examples concept representations $\bigcap S \in \mathcal{T}_{\mathcal{S}}$ are equivalently describable by concepts $C_{\mathcal{S}}$ due to $((\bigcap S)_{\mathcal{O}}^{\uparrow} = (K_{\mathcal{S}})_{\mathcal{O}}^{\uparrow})$, we will describe a SoS by the set of concepts $\{C_{\mathcal{S}} \mid S \in \mathcal{S}\}$. Sphere-based revision and contraction of concepts in an ontology are defined as follows.

Definition 2. Let $\mathcal{O} = \langle O, \mathcal{V}_p, \mathcal{V}_i \rangle$ be an ontology, \underline{C} be an \mathcal{O} -closed concept representation and D a concept from $\text{conc}((\mathcal{V}(\mathcal{O}) \cap \mathcal{V}_i) \cup \mathcal{V}_p)$. Furthermore let \mathcal{S} be a SoS for $[\underline{C}]^{\mathcal{O}}$ in \mathcal{O} . Then sphere-based revision of \mathcal{O} -closed concept representations $@_{\mathcal{S}}$ and sphere-based contraction of \mathcal{O} -closed concept representations $\ominus_{\mathcal{S}}$ are defined by $\underline{C} @_{\mathcal{S}} D = \bigcap (f_{\mathcal{S}}([D]^{\mathcal{O}}))$, $\underline{C} \ominus_{\mathcal{S}} D = (\underline{C} @_{\mathcal{S}} \neg D) \cap \underline{C}$, resp. Revision and contraction of single concepts are defined by $C @_{\mathcal{S}} D = C_{\mathcal{O}}^{\uparrow} @_{\mathcal{S}} D$, $C \ominus_{\mathcal{S}} D = C_{\mathcal{O}}^{\uparrow} \ominus_{\mathcal{S}} D$ resp.

As the \mathcal{O} -closure operator $(\cdot)_{\mathcal{O}}^{\uparrow}$ is a Tarskian consequence operator, one can prove that $@_{\mathcal{S}}$ and $\ominus_{\mathcal{S}}$ fulfill exactly those properties—adapted from sentences to concepts—that are fulfilled by the operators of [3].

4 Sphere-Based Reinterpretation

Using $\ominus_{\mathcal{S}}$ we now formally define sphere-based reinterpretation operators. Their input is an ontology and a trigger information, that is a *concept-based literal*, i.e. has the form $K(a)$ or $\neg K(a)$ for an atomic concept symbol K , for short: the form $\hat{K}(a)$. Their output is a new ontology. The input ontology \mathcal{O} is equipped with a family of many SoS: For all concept symbols $K \in \mathcal{V}_p$ there is a SoS $[K]^{\mathcal{O}}$, and a SoS $[\neg K]^{\mathcal{O}}$ for its negation.

Definition 3. A collection of systems of spheres \mathbb{S} of $\mathcal{O} = \langle O, \mathcal{V}_p, \mathcal{V}_i \rangle$ for concept-based literals over \mathcal{V}_p , for short $\langle \mathbb{S}(\hat{K}) \rangle_{\hat{K} \in CLit(\mathcal{V}_p)}$, is a family of SoS for each set of models $[\hat{K}]^{\mathcal{O}}$ of a concept literal \hat{K} over \mathcal{V}_p . A pair $\langle \mathcal{O}, \mathbb{S} \rangle$ of an ontology \mathcal{O} and a collection of SoS for \mathcal{O} is called structured ontology.

$\langle \mathcal{O}_1, \mathbb{S}^1 \rangle$ and $\langle \mathcal{O}_2, \mathbb{S}^2 \rangle$ are called equivalent, for short $\langle \mathcal{O}_1, \mathbb{S}^1 \rangle \cong \langle \mathcal{O}_2, \mathbb{S}^2 \rangle$ iff $\mathcal{O}_1 \equiv \mathcal{O}_2$ and additionally the collections of SoS are identical, $\mathbb{S}^1 = \mathbb{S}^2$.

The definition of sphere-based reinterpretation operators (Def. 4) relies on *weak operators* for reinterpretation defined in [2] as follows:

$$O \otimes K(a) = \begin{cases} O \cup \{K(a)\} & \text{if } O \cup \{K(a)\} \text{ is consistent,} \\ O_{[K/K']} \cup \{K(a), K' \sqsubseteq K\} & \text{else} \end{cases}$$

$$O \otimes \neg K(a) = \begin{cases} O \cup \{\neg K(a)\} & \text{if } O \cup \{\neg K(a)\} \text{ is consistent,} \\ O_{[K/K']} \cup \{\neg K(a), K \sqsubseteq K'\} & \text{else} \end{cases}$$

Definition 4. Let $\langle \mathcal{O}, \mathbb{S} \rangle$ be a structured ontology. Sphere-based reinterpretation $\odot_{\mathbb{S}}$ for concept-based literals is defined by

$$O \odot_{\mathbb{S}} \hat{K}(a) = \begin{cases} O \cup \{\hat{K}(a)\} & \text{if } O \cup \{\hat{K}(a)\} \text{ is consistent,} \\ O \otimes \hat{K}(a) \cup \{\hat{K} \sqsubseteq C \mid C \in (\hat{K} \ominus_{\mathbb{S}(\hat{K})} \neg msc_O(a))_{[K/K']}\} & \text{else} \end{cases}$$

The properties of these operators are listed in the following theorem. Some of the postulates have already been discussed by [1] for belief revision. Other postulates (such as the postulate RI-right preservation) are postulates that express desirable properties for semantic integration scenarios (see [5] for a discussion).

Theorem 1. For structured ontologies $\langle \mathcal{O}, \mathbb{S} \rangle$, $\langle \mathcal{O}_1, \mathbb{S}_1 \rangle$, and $\langle \mathcal{O}_2, \mathbb{S}_2 \rangle$ and concept-based literals α and β the following holds:

1. If $\langle \mathcal{O}_1, \mathbb{S}_1 \rangle \cong \langle \mathcal{O}_2, \mathbb{S}_2 \rangle$ then $(\mathcal{O}_1 \odot_{\mathbb{S}_1} \alpha) \equiv (\mathcal{O}_2 \odot_{\mathbb{S}_2} \alpha)$. (RI-left extensionality)
2. If $\alpha \equiv \beta$, then $(O \odot_{\mathbb{S}} \alpha) \equiv (O \odot_{\mathbb{S}} \beta)$. (RI-right extensionality)
3. $O \odot_{\mathbb{S}} \alpha = O \cup \{\alpha\}$ iff $O \cup \{\alpha\} \not\equiv \perp$. (RI-vacuity)
4. $\alpha \in O \odot_{\mathbb{S}} \alpha$ (RI-success)
5. There is a substitution σ s.t. $O\sigma \subseteq O \odot_{\mathbb{S}} \alpha$. (RI-left preservation)
6. There is a substitution σ s.t. $\alpha\sigma \in O \odot_{\mathbb{S}} \alpha$. (RI-right preservation)
7. There is a substitution σ s.t. $O \subseteq (O \odot_{\mathbb{S}} \alpha)\sigma$. (RI-left recoverability)
8. There is a substitution σ s.t. $\alpha \in (O \odot_{\mathbb{S}} \alpha)\sigma$. (RI-right recoverability)
9. $O \odot_{\mathbb{S}} \alpha \models \perp$ iff $O \models \perp$. (RI-consistency)

As collections of SoS \mathbb{S} are defined for a specific ontology \mathcal{O} , they are not necessarily also proper collections for the reinterpretation result $O \odot_{\mathbb{S}} \alpha$. In the following we mitigate this problem by proposing an iterated sphere-based reinterpretation operator $\odot_{\mathbb{S}}$. We require SoS to fulfill a condition called (SW) that strengthens the fourth condition on SoS according to Def. 1, requiring it to be well-ordered. Adapting the terminology of [7], we call a collection of systems of spheres *well-behaved* if it contains only well-ordered systems of spheres.

Let $O_{res} = O \odot_{\mathbb{S}} \alpha$ be the result of reinterpretation with $\alpha = \hat{K}(a)$ according to the one-step sphere-based reinterpretation. The main challenge in defining the follow-up sphere collection is the change of the vocabulary: some of the symbols of the receiver's ontology become private. In order to handle this vocabulary dynamics we use function $Dyn\mathcal{P}(M_{\mathcal{O}}) \rightarrow \mathcal{P}(M_{O_{res}})$; $S \mapsto Dyn(S) = \bigcup F[S_{[K/K']}] \cap M_{O_{res}}$ relying on the function F from Prop. 2.

Definition 5. For $\mathcal{O} = \langle \mathcal{O}, \mathcal{V}_p, \mathcal{V}_i \rangle$ and $\alpha = \hat{K}(a)$, let $\mathcal{V}_1 = (\mathcal{V}(O_{[K/K']}) \cap \mathcal{V}_i) \cup \mathcal{V}_p \setminus \{K\}$ and $\mathcal{V}_2 = (\mathcal{V}(O_{[K/K']}) \cap \mathcal{V}_i) \cup \mathcal{V}_p$ and $F = F_{\mathcal{O}, \mathcal{V}_1, \mathcal{V}_2}$ be the function $F(X) = \{Y \in M_{O_{res}, \mathcal{V}_2} \mid Y \supseteq X\}$ defined in Prop. 2. Further assume that \mathbb{S} is a well-behaved collection of SoS w.r.t. \mathcal{O} for concept-based literals over \mathcal{V}_p . The follow-up collection of spheres of $O_{res} = O \odot_{\mathbb{S}} \alpha$ is defined as follows: If $O \cup \{\alpha\} \not\equiv \perp$, then for all concept literals \hat{L} with $L \in \mathcal{V}_p$ the follow-up SoS is defined as $\mathbb{S}'(\hat{L}) = \{S \cap M_{O_{res}} \mid S \in \mathbb{S}(\hat{L})\}$. If $O \cup \{\alpha\} \models \perp$ and if $L \neq K$, then one sets $\mathbb{S}'(\hat{L}) = \{Dyn(S) \mid S \in \mathbb{S}(\hat{L})\}$. If $O \cup \{\alpha\} \models \perp$ and $L = K$, then: $\mathbb{S}'(\hat{K}) = \{[\hat{K}]^{O_{res}}\} \cup \{Dyn(S) \mid S \in \mathbb{S}(\hat{K})\}$ and $\mathbb{S}'(\hat{K}) = \{[\hat{K}]^{O_{res}}\} \cup \{Dyn(\hat{K} \ominus_{\mathbb{S}(\hat{K})})\}$

$\neg msc_O(a))\} \cup \{Dyn(S) \mid S \in \mathbb{S}(\hat{K}) \text{ and } Dyn(S) \supseteq Dyn(\hat{K} \ominus_{\mathbb{S}(\hat{K})} \neg msc_O(a))\}.$
 (Here we use the notation $\overline{\hat{K}} = \neg K$ if $\hat{K} = K$ and $\overline{\hat{K}} = K$ if $\hat{K} = \neg K$.)

The follow-up collection of systems of spheres \mathbb{S}' of Def. 5 are well-behaved.

An iterated operator is called *stable* [2] if after some step the outcomes of the operator do not change anymore—assuming that the set of triggers in the input sequence is finite. (Triggers may be sent more than once.) Sphere-based revision reinterpretation is strong in the sense that it does not forget about the reinterpretation history—and hence stability is not guaranteed.

Theorem 2. *Iterated sphere-based reinterpretation operators are not stable.*

For the proof one may use the same example as in [2, Theorem 7.15].

5 Conclusion

Following the general idea of reinterpretation for resolving conflicts caused by inter-ontological ambiguities, this paper defined iterable reinterpretation operators that rely on the preference structure of systems of spheres and showed (at least for the single-step case) that it fulfills some desirable properties.

Questions for future work are: What is a full characterization of iterated sphere-based reinterpretation operators via postulates? What is the best way to extend the approach to handle not only concept-based literals but also whole triggering ontologies—using still systems of spheres?

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